**MULTIPLE CRITERIA DECISION MAKING**

**by Simon Moss**

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| **Introduction** |

**Example of the issue**

Consider the following example. You need to decide which of five academics to choose as a supervisor. As the following table shows, these five academics, Adam, Betty, Carl, Donna, and Ernie, vary on five key characteristics or criteria:

* the number of years they have worked in academia
* the number of publications
* the number of research candidates they have supervised
* the number of hours they can dedicate to you each month
* the percentage of their students who had withdrawn before submitting their thesis

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 12 | 67 | 11 | 1 | 54 |
| Betty | 3 | 15 | 16 | 4 | 13 |
| Carl | 6 | 28 | 14 | 1 | 19 |
| Donna | 3 | 10 | 9 | 3 | 32 |
| Ernie | 16 | 28 | 6 | 1 | 41 |

 The decision is hard because the supervisors vary on more than one criteria. To illustrate, if you compare the first two supervisors, Adam and Betty, you will observe that

* Adam is more experienced and has published extensively—and, therefore, is knowledgeable
* Yet, Betty can dedicate more time to you; you are also unlikely to withdraw if she supervises you

The question, then, is how should you best weigh these considerations? How can you decide, for example, whether the experience of Adam outweighs the limited time or attention this supervisor might offer? To answer this question, you could apply some formula that combines all these columns into one column, called a preference score, as depicted in the final column of the following table. In this example

* the preference score of Betty is higher than is the preference score of the other supervisors
* so you would choose Betty as your supervisor

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage | Preference score |
| Adam | 12 | 67 | 11 | 1 | 54 | 3.9 |
| Betty | 3 | 15 | 16 | 4 | 13 | 5.3 |
| Carl | 6 | 28 | 14 | 1 | 19 | 2.8 |
| Donna | 3 | 10 | 9 | 3 | 32 | 1.9 |
| Ernie | 16 | 28 | 6 | 1 | 41 | 3.7 |

 But, what is the best formula to generate the preference score? How can you decide which formula to use? To answer this question, however, we first need to resolve some complications.

**Beneficial criteria versus cost criteria**

 The first complication revolves around a vital distinction between two classes of criteria. Return to the previous table and attempt to divide these criteria into two classes. Could you decipher the two classes?

The answer appears in the following table. In particular, as this table reveals, in one class, called beneficial criteria, are criteria in which high values are favourable. In the other class, called cost criteria, are criteria in which high values are unfavourable. These two classes need to be treated differently, as this document will later show.

|  |  |
| --- | --- |
| Beneficial criteria | Cost or non-beneficial criteria |
| High values are favourable | High values are unfavourable |
| In this example, the beneficial criteria are* years in academia
* number of publications
* number of supervised students, and
* hours a month of supervision
 | In this example, the cost criteria are* withdrawal percentage
 |
| To illustrate, a high value in years of academia indicates greater experience and is thus favourable | To illustrate, a high value in withdrawal percentage indicates that students are not always satisfied and is thus unfavourable |

**Normalisation and standardisation**

The second complication revolves around the observation that numbers are higher in some columns relative to other columns. In the previous example

* hours a month comprise small numbers, whereas number of publications comprise higher numbers
* the formulas you would use to combine these columns might be affected excessively by the criteria or columns with the higher numbers

To overcome this problem, you need to apply some adjustment, called normalisation or standardisation. After this normalisation or standardisation is completed, the magnitude and variability of these numbers are quite similar across the columns. To illustrate, you could

* for each beneficial criteria, divide each value by the maximum
* for each cost criteria, divide the minimum by each value.

To illustrate, the following table now presents the minimum and maximum value of each criteria or column. Therefore

* to normalize the number of years Adam has been employed in academia, you would divide 12 by 16, the maximum of this column
* to normalize the withdrawal percentage that corresponds to Adam, you would divide 13 by 54

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 12 | 67 | 11 | 1 | 54 |
| Betty | 3 | 15 | 16 | 4 | 13 |
| Carl | 6 | 28 | 14 | 1 | 19 |
| Donna | 3 | 10 | 9 | 3 | 32 |
| Ernie | 16 | 28 | 6 | 1 | 41 |
| Minimum | 3 | 10 | 6 | 1 | 13 |
| Maximum | 16 | 67 | 16 | 4 | 54 |

The following table presents the calculations. The next table presents the results of these calculations. As this table reveals, all the values are now between 0 and 1—and, therefore, have been normalised or standardised.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 12/16 | 67/67 | 11/16 | 1/4 | 13/54 |
| Betty | 3/16 | 15/67 | 16/16 | 4/4 | 13/13 |
| Carl | 6/16 | 28/67 | 14/16 | 1/4 | 13/19 |
| Donna | 3/16 | 10/67 | 9/16 | 3/4 | 13/32 |
| Ernie | 16/16 | 28/67 | 6/16 | 1/4 | 13/41 |
| Minimum | 3 | 10 | 6/16 | 1 | 13 |
| Maximum | 16 | 67 | 16 | 4 | 54 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 0.75 | 1.00 | 0.69 | 0.25 | 0.24 |
| Betty | 0.19 | 0.22 | 1.00 | 1.00 | 1.00 |
| Carl | 0.38 | 0.42 | 0.88 | 0.25 | 0.68 |
| Donna | 0.19 | 0.15 | 0.56 | 0.75 | 0.41 |
| Ernie | 1.00 | 0.42 | 0.38 | 0.25 | 0.32 |
| Minimum | 3 | 10 | 6 | 1 | 13 |
| Maximum | 16 | 67 | 16 | 4 | 54 |

**Weighting**

The third complication is that some criteria might be more important than other criteria. To demonstrate

* years in academia might not be especially important; people who have been academics over many years might be more experienced but might also be more jaded
* hours a month might be more important; supervisors who are too occupied with other responsibilities might not be able to offer enough guidance.

To weight the various criteria, you first need to

* assign a weight, between 0 and 1, to each column
* adjust the weights so they sum to 1

To illustrate, the weights for the five criteria might be

* years in academia: 0.1
* publications: 0.1
* supervised students: 0.2
* hours a month: 0.3
* withdrawal percentage: 0.3

These weights are somewhat arbitrary. Nevertheless, these weight do seem reasonable—because hours a month and withdrawal percentage are directed related to supervision. Furthermore, these weights sum to 1. Now, multiply each weight by the corresponding value.

* The following table displays the calculations
* The next table presents the results of these calculations

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |
| Adam | 0.75 x 0.1 | 1.00 x 0.1 | 0.69 x 0.2 | 0.25 x 0.3 | 0.24 x 0.3 |
| Betty | 0.19 x 0.1 | 0.22 x 0.1 | 1.00 x 0.2 | 1.00 x 0.3 | 1.00 x 0.3 |
| Carl | 0.38 x 0.1 | 0.42 x 0.1 | 0.88 x 0.2 | 0.25 x 0.3 | 0.68 x 0.3 |
| Donna | 0.19 x 0.1 | 0.15 x 0.1 | 0.56 x 0.2 | 0.75 x 0.3 | 0.41 x 0.3 |
| Ernie | 1.00 x 0.1 | 0.42 x 0.1 | 0.38 x 0.2 | 0.25 x 0.3 | 0.32 x 0.3 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |
| Adam | 0.08 | 0.10 | 0.14 | 0.08 | 0.07 |
| Betty | 0.02 | 0.02 | 0.20 | 0.30 | 0.30 |
| Carl | 0.04 | 0.04 | 0.18 | 0.08 | 0.20 |
| Donna | 0.02 | 0.02 | 0.11 | 0.23 | 0.12 |
| Ernie | 0.10 | 0.04 | 0.08 | 0.08 | 0.10 |

Finally, you merely need to sum the numbers that appear in each row, as illustrated in the following table. In short, to generate these preference scores, you

* assigned each criterion a weight
* multiplied each value by this weight
* summed these results

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage | Preference score |
| Adam | 0.08 | 0.10 | 0.14 | 0.08 | 0.07 | 0.46 |
| Betty | 0.02 | 0.02 | 0.20 | 0.30 | 0.30 | 0.84 |
| Carl | 0.04 | 0.04 | 0.18 | 0.08 | 0.20 | 0.54 |
| Donna | 0.02 | 0.02 | 0.11 | 0.23 | 0.12 | 0.49 |
| Ernie | 0.10 | 0.04 | 0.08 | 0.08 | 0.10 | 0.39 |

**Interpret the preference scores**

 To reach a conclusion, you merely need to identify the highest preference score. In this example, the highest preference score corresponds to Betty. So, you should choose Betty as your supervisor.

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| **Normalisation and standardisation** |

The previous example illustrated one variant of an approach called multiple criteria decision making or MCDM. Whenever you need to choose between options that vary on multiple criteria, such as years in academia and number of publications, this approach is informative.

Yet, researchers have developed many variants of this approach. Each variant is most applicable in particular circumstances. For example, these variants differ on

* how the original scores, called performance values, should be normalized
* how the researcher should convert these normalized performance values to preference scores
* how the weights should be estimated

This section outlines and illustrates the various formulas that researchers can apply to normalise the performance values or original scores. The following table outlines these formulas.

|  |  |  |
| --- | --- | --- |
| Name of approach | Formula or algorithm that applies to beneficial criteria | Formula or algorithm that applies to cost criteria |
| Linear normalisation I | * Divide each value you want to normalise by the maximum in this column
 | * Divide each value you want to normalize by the maximum in this column
* Then subtract this answer from 1
 |
| Linear normalisation II | * As above
 | * Divide the minimum in this column by the value you want to normalize
 |
| Linear normalisation: Max-min method | * Deduct the minimum in this column from the value you want to normalize
* Divide the answer by the difference between the maximum and minimum within this column
 | * Deduct the value you want to normalize from the maximum in this column
* Divide the answer by the difference between the maximum and minimum within this column
 |
| Linear normalisation: Sum method | * Divide the value you want to normalize from the sum of all the entries in this column
 | * First calculate the reciprocal of all the values in this column—that is, 1/(each value)
* Divide each reciprocal by the sum of these reciprocals.
* For example, consider the first value in a column.
* To normalize this value, you would divide the reciprocal of this value by the sum of these reciprocals
 |
| Vector normalisation | * Square all the values in this column
* Add the squared values for each column separately—called the sum of squared values
* Calculate the square root of this sum of squared values
* Divide the value you want to normalize by this square root of sum of squared values
 | * Complete the same approach as you would for beneficial criteria
* But subtract from 1
 |
| Enhanced accuracy normalisation | * First, subtract each value in the column from the maximum in this column
* Sum these values

Then, to normalise a specific value* Subtract this value from the maximum
* Divide the answer by the sum that was calculated in the previous step
 | * First, subtract the minimum of a column from each value in this column
* Sum these values

Then, to normalise a specific value* Subtract the minimum in this column from the value
* Divide the answer by the sum that was calculated in the previous step
 |
| Logarithmic normalisation  | * Compute the natural log of each value in a column.
* For example, in Excel, you could enter something like =LN(25) to compute the natural log of 25
* Calculate the product of all these values; that is, multiplu these logged values

Then, to normalise a specific value* Divide the natural log of this value by the product you just calculated
 | * Complete the same approach as you would for beneficial criteria
* Subtract this answer from 1
* Then divide this answer by m – 1, where m is the number of options you want to compare—such as number of supervisors
* Finally, subtract this answer from 1
 |

**Illustration of these formulas**

 To illustrate these normalisation formulas, consider the following table of data. Now suppose you want to apply one of the techniques, called vector normalisation, to normalise these data—to convert these numbers to values that range between 0 and 1.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 12 | 67 | 11 | 1 | 54 |
| Betty | 3 | 15 | 16 | 4 | 13 |
| Carl | 6 | 28 | 14 | 1 | 19 |
| Donna | 3 | 10 | 9 | 3 | 32 |
| Ernie | 16 | 28 | 6 | 1 | 41 |

According to the relevant formula, the first step is to square all the performance values. The following table displays these squared values

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 144 | 4489 | 121 | 1 | 2916 |
| Betty | 9 | 225 | 256 | 16 | 169 |
| Carl | 36 | 784 | 196 | 1 | 361 |
| Donna | 9 | 100 | 81 | 9 | 1024 |
| Ernie | 256 | 784 | 36 | 1 | 1681 |

 The second step is to sum the squared performance values in each column. The bottom row in the following table reveals these sums.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 144 | 4489 | 121 | 1 | 2916 |
| Betty | 9 | 225 | 256 | 16 | 169 |
| Carl | 36 | 784 | 196 | 1 | 361 |
| Donna | 9 | 100 | 81 | 9 | 1024 |
| Ernie | 256 | 784 | 36 | 1 | 1681 |
| Sum of squares | 454 | 6382 | 690 | 28 | 6151 |
| Square root of sum of squares | 21.31 | 79.89 | 26.27 | 5.29 | 78.43 |

 The third step is to divide each of the original values by the sum of squares in the corresponding column. The following table presents the calculations to illustrate this step. The next table then presents the results of these calculations. As this table shows, all the values are between 0 and 1.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 12/21.31 | 67/79.89 | 11/26.27 | 1/5.29 | 54/78.43 |
| Betty | 3/21.31 | 15/79.89 | 16/26.27 | 4/5.29 | 13/78.43 |
| Carl | 6/21.31 | 28/79.89 | 14/26.27 | 1/5.29 | 19/78.43 |
| Donna | 3/21.31 | 10/79.89 | 9/26.27 | 3/5.29 | 32/78.43 |
| Ernie | 16/21.31 | 28/79.89 | 6/26.27 | 1/5.29 | 41/78.43 |
| Sum of squares | 454 | 6382 | 690 | 28 | 6151 |
| Square root of sum of squares | 21.31 | 79.89 | 26.27 | 5.29 | 78.43 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 0.56 | 0.84 | 0.42 | 0.19 | 0.69 |
| Betty | 0.14 | 0.19 | 0.61 | 0.76 | 0.17 |
| Carl | 0.28 | 0.35 | 0.53 | 0.19 | 0.24 |
| Donna | 0.14 | 0.13 | 0.34 | 0.57 | 0.41 |
| Ernie | 0.75 | 0.35 | 0.23 | 0.19 | 0.52 |
| Sum of squares | 454 | 6382 | 690 | 28 | 6151 |
| Square root of sum of squares | 21.31 | 79.89 | 26.27 | 5.29 | 78.43 |

 Finally, for the cost criteria—withdrawal percentage—subtract the values from 1. The results appear in the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 0.56 | 0.84 | 0.42 | 0.19 | 0.31 |
| Betty | 0.14 | 0.19 | 0.61 | 0.76 | 0.83 |
| Carl | 0.28 | 0.35 | 0.53 | 0.19 | 0.76 |
| Donna | 0.14 | 0.13 | 0.34 | 0.57 | 0.59 |
| Ernie | 0.75 | 0.35 | 0.23 | 0.19 | 0.48 |
| Sum of squares | 454 | 6382 | 690 | 28 | 6151 |
| Square root of sum of squares | 21.31 | 79.89 | 26.27 | 5.29 | 78.43 |

Admittedly, this example illustrates only one of the techniques. Usually, after you practice one technique, your capacity to understand and learn the other techniques improves. These methods are quite straightforward in practice.

**Which normalisation technique should I use?**

 To conduct multiple criteria decision making, you need to apply only one of these techniques at a time. So, which normalisation technique should you use? Is one technique better than another technique? To answer this question, apply the following principles:

* some researchers suggest you simply use the linear normalization formula—especially the second formula that appeared in the previous table
* this formula seems to distinguish or separate the performance values to a significant extent
* nevertheless, which formula you apply should partly depend on how you blend the normalized performance values to generate the preference score—as discussed later.

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| **Weighted sum model and weighted product model** |

 Once you normalise the original scores or performance values, your next task is to convert the normalised performance values to preference scores. Essentially, you want to combine all the criteria into one value. Although you can apply a variety of approaches, two of the simplest techniques are

* the weighted sum model or WSM and
* weighted product model WPM

The weighted sum model was illustrated towards the beginning of this document. That is, multiplying the normalised values by the weights—and then summing these products—is called the weighted sum model.

The weighted product model is slightly different. First, like the weighted sum model, you need to assign weights to each criterion. These weights must sum to one, as illustrated in the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |
| Adam | 0.56 | 0.84 | 0.42 | 0.19 | 0.31 |
| Betty | 0.14 | 0.19 | 0.61 | 0.76 | 0.83 |
| Carl | 0.28 | 0.35 | 0.53 | 0.19 | 0.76 |
| Donna | 0.14 | 0.13 | 0.34 | 0.57 | 0.59 |
| Ernie | 0.75 | 0.35 | 0.23 | 0.19 | 0.48 |

Second, calculate the normalised value to the power of this weight. If this description is ambiguous, simply scan the following table—a table that displays these calculations—and you will understand what I mean. The next table then presents the results of these calculations

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |
| Adam | 0.560.1 | 0.840.1 | 0.420.2 | 0.190.3 | 0.310.3 |
| Betty | 0.140.1 | 0.190.1 | 0.610.2 | 0.760.3 | 0.830.3 |
| Carl | 0.280.1 | 0.350.1 | 0.530.2 | 0.190.3 | 0.760.3 |
| Donna | 0.140.1 | 0.130.1 | 0.340.2 | 0.570.3 | 0.590.3 |
| Ernie | 0.750.1 | 0.350.1 | 0.230.2 | 0.190.3 | 0.480.3 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |
| Adam | 0.94 | 0.98 | 0.84 | 0.61 | 0.70 |
| Betty | 0.82 | 0.85 | 0.91 | 0.92 | 0.95 |
| Carl | 0.88 | 0.90 | 0.88 | 0.61 | 0.92 |
| Donna | 0.82 | 0.82 | 0.81 | 0.84 | 0.85 |
| Ernie | 0.97 | 0.90 | 0.75 | 0.61 | 0.80 |

Finally, for each row—in this instance, supervisor—multiply these answers together. For Adam, you would multiply .94, .98, .84, .61, and .70. The results appear in the following table. According to this table

* Betty generates the highest preference score
* thus, Betty should be the preferred supervisor

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage | Preference score |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |  |
| Adam | 0.70 | 0.63 | 0.46 | 0.38 | 1.00 | 0.33 |
| Betty | 0.63 | 0.00 | 0.46 | 0.55 | 1.00 | 0.55 |
| Carl | 0.63 | 0.00 | 0.46 | 0.38 | 1.00 | 0.39 |
| Donna | 0.63 | 0.00 | 0.40 | 0.52 | 1.00 | 0.39 |
| Ernie | 0.72 | 0.00 | 0.40 | 0.38 | 1.00 | 0.32 |

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| **Weighted aggregate sum product assessment** |

 The previous section outlined two techniques that researchers often utilise to convert several columns of normalised performance values to a single column, called a preference score: the weighted sum model or WSM and the weighted product model or WPM. The obvious question, then, is which model should you apply? One common answer is to blend these two methods. First, you apply both methods to generate two columns of preference scores, as illustrated in the following table.

|  |  |  |
| --- | --- | --- |
| Name of supervisor | Preference score from the weighted sum model | Preference score from the weighted product model |
| Adam | 0.46 | 0.33 |
| Betty | 0.84 | 0.55 |
| Carl | 0.54 | 0.39 |
| Donna | 0.49 | 0.39 |
| Ernie | 0.39 | 0.32 |

 You then subject these preference scores to a formula that mirrors the following equation

|  |
| --- |
| Final preference score = 0.4 x preference score from WSM + 0.6 x preference score from WPM |

To illustrate

* for Adam, the final preference scores = 0.4 x 0.46 + 0.6 x 0.33 = .38, as the following table shows
* in this example, Betty generates the highest score and should thus be chosen

|  |  |  |  |
| --- | --- | --- | --- |
| Name of supervisor | Preference score from the weighted sum model | Preference score from the weighted product model | Final preference score |
| Adam | 0.46 | 0.33 | 0.38 |
| Betty | 0.84 | 0.55 | 0.67 |
| Carl | 0.54 | 0.39 | 0.45 |
| Donna | 0.49 | 0.39 | 0.43 |
| Ernie | 0.39 | 0.32 | 0.35 |

In this example, the preference score for the WSM was multiplied by 0.4. However, researchers can adjust this number, called or lambda. Usually, however, this  is usually close to 0.5.

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| **Technique for order preference by similarity to ideal solution or TOPSIS** |

 Thus far, to convert the normalised performance values to the preference score, all the techniques have involved weighted sums, products, or a combination of both. Yet, researchers have developed many other techniques to convert normalised performance values to the preference scores—and some of these techniques are more useful in specific circumstances. This section illustrates one of these approaches: technique for order preference by similarity to ideal solution or TOPSIS. To conduct this technique, first subject the original data to vector normalisation. The following table presents these normalised data.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 0.56 | 0.84 | 0.42 | 0.19 | 0.31 |
| Betty | 0.14 | 0.19 | 0.61 | 0.76 | 0.83 |
| Carl | 0.28 | 0.35 | 0.53 | 0.19 | 0.76 |
| Donna | 0.14 | 0.13 | 0.34 | 0.57 | 0.59 |
| Ernie | 0.75 | 0.35 | 0.23 | 0.19 | 0.48 |

 Second, as with the previous techniques, weight each criterion. These weights should sum to 1, as illustrated in the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |
| Adam | 0.56 | 0.84 | 0.42 | 0.19 | 0.31 |
| Betty | 0.14 | 0.19 | 0.61 | 0.76 | 0.83 |
| Carl | 0.28 | 0.35 | 0.53 | 0.19 | 0.76 |
| Donna | 0.14 | 0.13 | 0.34 | 0.57 | 0.59 |
| Ernie | 0.75 | 0.35 | 0.23 | 0.19 | 0.48 |

 Third, multiply each of the normalised performance values by the weight in this column. The following table illustrates the calculation. The next table presents the results of this calculation. This table is called the weighted normalized decision matrix

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |
| Adam | 0.56 x 0.1 | 0.84 x 0.1 | 0.42 x 0.2 | 0.19 x 0.3 | 0.31 x 0.3 |
| Betty | 0.14 x 0.1 | 0.19 x 0.1 | 0.61 x 0.2 | 0.76 x 0.3 | 0.83 x 0.3 |
| Carl | 0.28 x 0.1 | 0.35 x 0.1 | 0.53 x 0.2 | 0.19 x 0.3 | 0.76 x 0.3 |
| Donna | 0.14 x 0.1 | 0.13 x 0.1 | 0.34 x 0.2 | 0.57 x 0.3 | 0.59 x 0.3 |
| Ernie | 0.75 x 0.1 | 0.35 x 0.1 | 0.23 x 0.2 | 0.19 x 0.3 | 0.48 x 0.3 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Weight | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 |
| Adam | 0.06 | 0.08 | 0.08 | 0.06 | 0.09 |
| Betty | 0.01 | 0.02 | 0.12 | 0.23 | 0.25 |
| Carl | 0.03 | 0.04 | 0.11 | 0.06 | 0.23 |
| Donna | 0.01 | 0.01 | 0.07 | 0.17 | 0.18 |
| Ernie | 0.08 | 0.04 | 0.05 | 0.06 | 0.14 |

 Fourth, for each criteria, record the maximum and minimum values as illustrated in the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | 0.06 | 0.08 | 0.08 | 0.06 | 0.09 |
| Betty | 0.01 | 0.02 | 0.12 | 0.23 | 0.25 |
| Carl | 0.03 | 0.04 | 0.11 | 0.06 | 0.23 |
| Donna | 0.01 | 0.01 | 0.07 | 0.17 | 0.18 |
| Ernie | 0.08 | 0.04 | 0.05 | 0.06 | 0.14 |
| Maximum | 0.08 | 0.08 | 0.12 | 0.23 | 0.25 |
| Minimum | 0.01 | 0.01 | 0.05 | 0.06 | 0.09 |

 Fifth, calculate what is called the Euclidean distance from the maximum. In particular, for each normalised value

* compute the difference between this value and the maximum
* square this difference
* for each supervisor you want to compare, sum these squares

The following table displays these calculations—in particular, the difference between this value and the maximum and the square. The next table then presents the results of these calculations as well as the sum of these squared differences.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Adam | (0.06-.08)2 | (0.08-.08)2 | (0.08-.12)2 | (0.06-.23)2 | (0.09-.25)2 |
| Betty | (0.01-.08)2 | (0.02-.08)2 | (0.12-.12)2 | (0.23-.23)2 | (0.25-.25)2 |
| Carl | (0.03-.08)2 | (0.04-.08)2 | (0.11-.12)2 | (0.06-.23)2 | (0.23-.25)2 |
| Donna | (0.01-.08)2 | (0.01-.08)2 | (0.07-.12)2 | (0.17-.23)2 | (0.18-.25)2 |
| Ernie | (0.08-.08)2 | (0.04-.08)2 | (0.05-.12)2 | (0.06-.23)2 | (0.14-.25)2 |
| Maximum | 0.08 | 0.08 | 0.12 | 0.23 | 0.25 |
| Minimum | 0.01 | 0.01 | 0.05 | 0.06 | 0.09 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage | Sum of squares from maximum |
| Adam | 0.000 | 0.000 | 0.002 | 0.029 | 0.026 | 0.057 |
| Betty | 0.005 | 0.004 | 0.000 | 0.000 | 0.000 | 0.009 |
| Carl | 0.003 | 0.002 | 0.000 | 0.029 | 0.000 | 0.034 |
| Donna | 0.005 | 0.005 | 0.003 | 0.004 | 0.005 | 0.022 |
| Ernie | 0.000 | 0.002 | 0.005 | 0.029 | 0.012 | 0.048 |
| Maximum | 0.08 | 0.08 | 0.12 | 0.23 | 0.25 | 0.25 |
| Minimum | 0.01 | 0.01 | 0.05 | 0.06 | 0.09 | 0.09 |

 Sixth, repeat this approach, but instead compute the Euclidean distance from the minimum. That is,

* compute the difference between each value and the minimum
* square this difference
* for each column, sum these squares. The results appear in the following table

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Name of supervisor | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage | Sum of squares from minimum |
| Adam | 0.003 | 0.005 | 0.001 | 0.000 | 0.000 | 0.008 |
| Betty | 0.000 | 0.000 | 0.005 | 0.029 | 0.026 | 0.060 |
| Carl | 0.000 | 0.001 | 0.004 | 0.000 | 0.020 | 0.025 |
| Donna | 0.000 | 0.000 | 0.000 | 0.012 | 0.008 | 0.021 |
| Ernie | 0.005 | 0.001 | 0.000 | 0.000 | 0.003 | 0.008 |
| Maximum | 0.08 | 0.08 | 0.12 | 0.23 | 0.25 | 0.25 |
| Minimum | 0.01 | 0.01 | 0.05 | 0.06 | 0.09 | 0.09 |

 Finally, to generate an overall preference score, subject these sums of square to minimums and maximums to a simple formula. Specifically, for each supervisor, the formula is

* preference score = minimum sum of square / (sum of minimum and maximum sum of square)

The following table illustrates this calculation. According to this calculation, Betty should be chosen. Again, she generates the highest preference score.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Name of supervisor | Sum of squares from maximum | Sum of squares from minimum | Calculation | Final preference score |
| Adam | 0.057 | 0.008 | 0.008/(0.008+0.057) | 0.123 |
| Betty | 0.009 | 0.060 | 0.06/(0.06+0.009) | 0.870 |
| Carl | 0.034 | 0.025 | 0.025/(0.025+0.034) | 0.424 |
| Donna | 0.022 | 0.021 | 0.021/(0.021+0.022) | 0.488 |
| Ernie | 0.048 | 0.008 | 00.008/(0.008+0.048) | 0.143 |

|  |
| --- |
| **How to estimate suitable weights** |

**Construct a matrix**

 So far, this document has illustrated a few techniques researchers can apply to convert normalised performance values to a preference score. For each technique, the researcher needed to weight the various criteria. The key question, however, is which approaches or principles should researchers use to weight these criteria appropriate? Several approaches, such as the analytic hierarchy process or AHP, could be considered.

To apply AHP, you need to complete a series of simple activities. First, construct a table, called a pairwise comparison matrix, in which

* each column and each row corresponds to one criterion, as the following display illustrates
* the diagonals equal 1

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Years in academia | 1  |   |   |   |   |
| Publications |   | 1  |   |   |   |
| Supervised students |   |   | 1  |   |   |
| Hours a month |   |   |   | 1  |   |
| Withdrawal percentage |   |   |   |   | 1  |

**Estimate the values in this matrix**

 In each cell, include numbers to represent the extent to which you feel one criterion is more important than another criterion. To illustrate, consider the following matrix. To interpret this matrix

* numbers that exceed 1 indicate the criterion in this row is more important than is the criterion in this column
* numbers that are less than 1 indicate the criterion in this row is less important than is the criterion in this column

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Years in academia | 1  | **1/2**  | 1/3 | 1/5  | 1/4  |
| Publications | **2**  | 1  | 1/2  | 1/4  | 1/3  |
| Supervised students | 3 | 2 | 1  | 1/2  | 1/2  |
| Hours a month | 5  |  4 | 2 | 1  | 1  |
| Withdrawal percentage | 4  |  3  | 2 | 1  | 1  |

To illustrate, in this example

* publications are more important than years in academia but less important than supervised students, hours a month, or withdrawal
* similarly, supervised students is more important than years in academia and publications but less important than hours a month and withdrawal percentage

The magnitude of these numbers are important as well. Typically, these numbers vary from 1 to 9. For example

* a 3 would indicate the criterion in the row is **moderately** more important than is the criterion in the column
* a 6 would indicate the criterion in the row is **appreciably** more important than is the criterion in the column
* a 9 would indicate the criterion in the row is **extraordinarily** more important than is the criterion in the column

Finally, in this example, you may have observed how some numbers are reciprocals of other numbers. For example

* the bold number in the first column indicates that publications seem twice as important than years in academia
* the bold number in the second column is the inverse or reciprocal, implying that years in academia is half as important than publications

So, to complete these tables, you should initially confine your attention to the cells below the diagonal. That is, using your knowledge about these criteria, you would

* estimate the extent to which the second criterion is more important than is the first criterion
* estimate the extent to which the third criterion is more important than is the first criterion, and so forth
* to complete the entries above the diagonal, calculate the inverse or reciprocal of the corresponding numbers below the diagonal.

**Calculate the criterion weights**

 After you enter the numbers in these cells, and perhaps calculate the fractions, you should then sum each column. The results appear in the following table

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Years in academia | 1  | 0.5  | 0.333 | 0.2  | 0.25  |
| Publications | 2  | 1  | 0.5  | 0.25  | 0.33  |
| Supervised students | 3 | 2 | 1  | 0.5  | 0.5  |
| Hours a month | 5  |  4 | 2 | 1  | 1  |
| Withdrawal percentage | 4  |  3  | 2 | 1  | 1  |
| Sum | 15.000 | 10.500 | 5.833 | 2.950 | 3.080 |

 Next

* divide each of the values in this table by this sum
* finally, average the values in each row, as the following table shows, to generate an index called the criterion weights.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage | Criterion weights |
| Years in academia | 0.067 | 0.048 | 0.057 | 0.068 | 0.081 | 0.064 |
| Publications | 0.133 | 0.095 | 0.086 | 0.085 | 0.107 | 0.101 |
| Supervised students | 0.200 | 0.190 | 0.171 | 0.169 | 0.162 | 0.179 |
| Hours a month | 0.333 | 0.381 | 0.343 | 0.339 | 0.325 | 0.344 |
| Withdrawal percentage | 0.267 | 0.286 | 0.343 | 0.339 | 0.325 | 0.312 |
| Sum | 15.000 | 10.500 | 5.833 | 2.950 | 3.080 |  |

 For example, in this instance

* the first value in the first column, 0.067, equals 1, derived from the previous table, divided by 15
* the criterion weight in the first row is the average of the numbers in this row: 0.067, 0.048, 0.057, 0.068, and 0.081
* as the final column indicates, the weights you should use to combine the columns into one preference score are 0.064, 0.101 0.179, 0.344, and 0.312 for the five criteria respectively.

**Assess consistency**

 These weights are useful, provided you estimated the relative importance of these criteria appropriately. Researchers sometimes, however, estimate these values somewhat inconsistently or randomly. For example, they might estimate that

* Criterion 1 is double as important as Criterion 2
* Criterion 2 is double as important as Criterion 3 but
* Criterion 3 is double as important as Criterion 1.

These estimates seem implausible and inconsistent. If Criterion 1 is more important than Criterion 2, and Criterion 2 is more important than Criterion 3, you would assume that Criterion 1 would be more important than Criterion 3 as well.

Fortunately, researchers have uncovered a technique that you can use to assess consistency. First, multiply the original values in the pairwise comparison matrix by the criterion weight in this column. The following table displays the calculations. The next table then presents the results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Criterion weight | 0.064 | 0.101 | 0.179 | 0.344 | 0.312 |
| Years in academia | 1 x .064 | 0.5 x .101 | 0.333 x 0.179 | 0.2 x 0.344 | 0.25 x 0.312  |
| Publications | 2 x .064 | 1 x .101 | 0.5 x 0.179 | 0.25 x 0.344 | 0.33 x 0.312  |
| Supervised students | 3 x .064 | 2 x .101 | 1 x 0.179 | 0.5 x 0.344 | 0.5 x 0.312  |
| Hours a month | 5 x .064 |  4 x .101 | 2 x 0.179 | 1 x 0.344 | 1 x 0.312  |
| Withdrawal percentage | 4 x .064  |  3 x .101 | 2 x 0.179 | 1 x 0.344  | 1 x 0.312  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |
| Criterion weight | 0.064 | 0.101 | 0.179 | 0.344 | 0.312 |
| Years in academia | 0.064 | 0.051 | 0.060 | 0.069 | 0.078 |
| Publications | 0.128 | 0.101 | 0.090 | 0.086 | 0.103 |
| Supervised students | 0.192 | 0.202 | 0.179 | 0.172 | 0.156 |
| Hours a month | 0.320 | 0.404 | 0.358 | 0.344 | 0.312 |
| Withdrawal percentage | 0.256 | 0.303 | 0.358 | 0.344 | 0.312 |

Then, merely sum the values in each row to generate a column called the weighted sum value. The following table reveals the results

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Years in academia | Publications | Supervised students | Hours a month | Withdrawal percentage |  |
| Criterion weight | 0.064 | 0.101 | 0.179 | 0.344 | 0.312 | Weighted sum value |
| Years in academia | 0.064 | 0.051 | 0.060 | 0.069 | 0.078 | 0.321 |
| Publications | 0.128 | 0.101 | 0.090 | 0.086 | 0.103 | 0.507 |
| Supervised students | 0.192 | 0.202 | 0.179 | 0.172 | 0.156 | 0.901 |
| Hours a month | 0.320 | 0.404 | 0.358 | 0.344 | 0.312 | 1.738 |
| Withdrawal percentage | 0.256 | 0.303 | 0.358 | 0.344 | 0.312 | 1.573 |

 Next, for each row or criterion, divide the weighted sum value by the criterion weight. This calculation will generate five numbers, as shown in the final column in the following table. Then, calculate the average of these five numbers, called  max.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Weighted sum value | Criterion weight | Weighted sum value criterion weight | Result of this calculation |
| Years in academia | 0.321 | 0.064 | 0.321/0.064 | 5.016 |
| Publications | 0.507 | 0.101 | 0.507/0.101 | 5.020 |
| Supervised students | 0.901 | 0.179 | 0.901/0.179 | 5.034 |
| Hours a month | 1.738 | 0.344 | 1.738/0.344 | 5.052 |
| Withdrawal percentage | 1.573 | 0.312 | 1.573/0.312 | 5.042 |
|  |  |  |  |  max = 5.033 |

 Finally, enter this  max into the following formula, in which n is the number of criteria. In this instance, the Consistency Index = (5.033 – 5) / (5 – 1) = .00825

|  |
| --- |
| Consistency Index = ( max – n) / (n – 1) |

 So, how do we interpret this .00825? Does this value indicate the pairwise comparisons were consistent or not? To answer this question, we need to

* extract the random index from the following table—an estimate of how large the consistency index would have been had we entered random numbers in the pairwise comparison matrix. In this instance, the Random Index is 0.90 because the number of criteria is 4
* divide the Consistency Index by the Random index. In this instance, .00825/0.90 = .009
* whenever this ratio is less than .10, the pairwise comparison matrix is regarded as consistent

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Random Index | 0.00 | 0.00 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 |

 Hence, in this instance, the pairwise comparison matrix is consistent. The values we entered were plausible rather than incompatible with each other. Consequently, we can utilise the criterion weights we generated in subsequent analyses.

|  |
| --- |
| **Local weights and global weights** |

 The previous examples revolved around five criteria: years in academia, publications, supervised students, hours a month, and withdrawal percentage. Sometimes, the criteria comprise sub-criteria. For example, publications might comprise two sub-criteria

* number of papers
* number of books

In these circumstances, you need to choose one of two approaches. First, you could replace the criterion with these sub-criteria. That is, you could supplant number of publications with two criteria—number of papers and number of books—and thus proceed with six criteria instead of five criteria. If you apply this approach, however, you might experience a problem. In particular, when constructing the pairwise comparison matrix to estimate the weights

* you might be able to compare number of publications to the other criteria
* but you might not be able to compare the sub-criteria—number of papers and number of books—with the other criteria as readily
* that is, the sub-criteria may be too specific; therefore, these sub-criteria might seem too different to the other criteria to compare meaningfully.

Instead, you might consider a second approach. In particular, you could first, calculate the weights for each criterion. For example, as demonstrated in the previous section, the weights that correspond to the five criteria might be 0.064, 0.101, 0.179, 0.344, and 0.312. You could then calculate the weights for the sub-criteria of each criterion. That is, you would

* begin with a pairwise comparison matrix that corresponds to the sub-criteria of one criterion—as the following table shows
* then compute the weights of each sub-criterion, using the principles that were delineated in the previous section
* the weights that correspond to the two sub-criteria might be 0.7 and 0.3 for example, called local weights

|  |  |  |
| --- | --- | --- |
|  | Number of papers | Number of books |
| Number of papers | 1 | 0.5 |
| Number of books | 2 | 1 |

Finally, you would multiply these local weights—0.7 and 0.3—with the previous weight for this criterion—.101. Thus

* the final weight for number of papers would be 0.7 x 0.101
* the final weight for number of books would be .0.3 x .101
* you would then use these sub-criteria, instead of the broader criterion, in subsequent analyses.

|  |
| --- |
| **How to undertake multiple criteria decision making more efficiently** |

 Thus far, this document has discussed a range of techniques that can facilitate decisions. These techniques, although simple in principle, can be cumbersome in practice. To complete the calculations, you could

* use Excel, especially if your Excel skills are reasonably advanced
* use the statistical package R. If you are not familiar with R, read the introductory notes on this software first. You could then visit <https://cran.r-project.org/web/packages/MCDA/MCDA.pdf> to learn how to conduct multiple criteria decision analysis in R.

|  |
| --- |
| **Further information** |

 To learn more about multiple criteria decision making, in YouTube, search “Manoj Mathew multiple criteria”. This channel includes other techniques, such as

* Promethee I and II or EDAS, designed to convert normalized values to preference scores
* the Entropy method or CRITIC method, designed to estimate suitable values when experts do not agree with one another
* fuzzy analytic hierarchy process, similar to AHP, except you do not need to specify precise values in the pairwise comparison matrix but a fuzzy interval instead

In addition, you could

* google “Multiobjective Combinatorial Optimization”—a variant of multiple criteria decision making that is applied when the original values are integers
* google other concepts such as outranking relations and progressive articulation of preferences